Some Details of Developing Slugs in Horizontal Two-Phase Flow

A photographic study has been carried out to uncover the details of development of slugs in horizontal two-phase flow. Based on the observations, it is proposed that the slugs originate as a consequence of local Kelvin-Helmholtz instability at the wave crest, rather than the instability of the whole wave. The findings are in reasonable agreement with published results on the transition to slug flow.

EUGENE KORDYBAN

University of Detroit
Detroit, MI 48221

SCOPE

It is generally accepted that slugs form in horizontal gas-liquid flow as a consequence of instability of the interfacial waves. Previous experimental studies, however, indicate certain discrepancies between the theory and actual conditions under which slug formation begins. The gas velocity is lower than that required to produce the instability of the wave, and there is no retardation of the waves as predicted by the theory.

The present work was undertaken to study the details of slug formation photographically with the purpose of uncovering physical reasons for the above discrepancies. Single long waves produced in a rectangular channel, were followed by a motion picture camera along the channel until a slug formed. In addition, the local instability of the liquid surface has been examined analytically.

CONCLUSIONS AND SIGNIFICANCE

The main visual observation of these experiments is that prior to slug formation small waves always appear on the crest of the large waves, and that a slug results from the rapid growth of one of these wavelets.

It is proposed that the appearance of these wavelets represents the onset of the Kelvin-Helmholtz instability. Thus slugs form as a result of local instability at the wave crest rather than the instability of the whole wave. The conditions under which the slugs formed in these experiments are in agreement with generally accepted criteria, so this does not appear to be a special case of slug formation.

The results of this work provide a physical basis for a number of assumptions of previous investigators.

INTRODUCTION

The slug flow pattern was observed early in the investigation of two-phase flows, and the conditions under which it occurs were determined empirically (Baker, 1954). More recently, Kordyban and Ranov (1970) and Kordyban (1977) have proposed that slugs develop as a result of Kelvin-Helmholtz instability of the waves enhanced by the presence of the top wall in the channel. Taitel and Dukler (1976) arrived at similar conclusions, while Wallis and Dobson (1973) regarded the water in the wave as an inverse of Benjamin's long bubble held stationary by the gas flow. Gardner (1979) examined this problem from the standpoint of energy transfer.

The Kelvin-Helmholtz instability may be interpreted physically as the instability of the wave which occurs when the low pressure at the crest, resulting from higher gas velocity there, overcomes the stabilizing effect of gravity. It is found by examining the pertinent wave equations that at the point of instability, wave celerity with respect to liquid current vanishes for low gas densities. The measured pressure at the wave surface (Kordyban, 1973, 1980) is close to but somewhat below that required by the theory. The waves of zero celerity, however, were never observed in the experiment (Kordyban, 1977; Gardner, 1979). Thus, this aspect of the Kelvin-Helmholtz has remained unexplained.

In the present work the development of slugs was studied photographically with the purpose of obtaining the physical details of this instability.

KELVIN-HELMHOLTZ INSTABILITY AND THE GROWTH OF WAVES

The gas stream flowing over a wave liquid surface affects the waves as a result of the presence of tangential forces—i.e., shear—and of normal forces which develop due to variation of pressure in the flowing gas. Many investigators consider the effects of shear to be secondary; consequently, only the forces due to pressure variations will be considered here.

The aerodynamic pressure variations in the gas bear a definite phase relationship to the liquid wave and may be considered to consist of a component 180° out of phase with the wave and a component in phase with the wave slope. If the former component is considered alone, its effect may best be illustrated if the wave is written in the form:

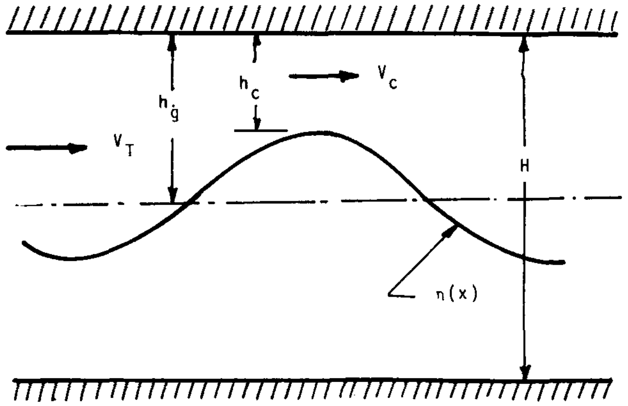

Figure 1. Flow configuration and dimensions.

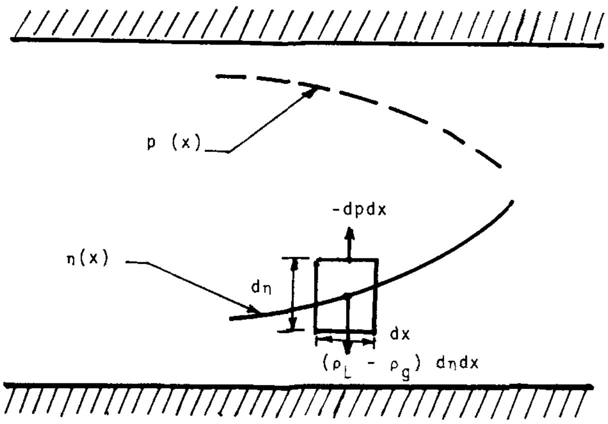

Figure 2. Forces on a liquid element.

$$\frac{\rho_1}{k}\frac{\partial^2\eta}{\partial t^2} + (\rho_1 g + T_1 k^2)\eta = -P_g \tag{1}$$

Expressing the pressure in gas as:

$$P_g = -\alpha_g \eta$$

Eq. 1 becomes:

$$\frac{\rho_1}{k}\frac{\partial^2\eta}{\partial t^2} + (\rho_1 g + T_1 k^2 - \alpha_g)\eta = 0 \tag{2}$$

Equation 2 has a periodic form of solution in time for $\alpha_g < \rho_1 g + T_1 k^2$ while for $\alpha_g > \rho_1 g + T_1 k^2$, the solution becomes exponential. Thus the point at which $\alpha_g = \rho_1 g + T_1 k^2$ represents the onset of instability, which is known as Kelvin-Helmholtz instability.

Physically, the Kelvin-Helmholtz instability may be interpreted as occurring at a point where aerodynamic suction at the crest exceeds the stabilizing effect of gravity. Also, since for small waves the celerity is given by

$$c = \left(\frac{g}{k} + \frac{T_1 k}{\rho_1} - \frac{\alpha}{\rho_1 k}\right)^{1/2},$$

at the point of instability $c = 0$ and the waves should become stationary on liquid at rest, but would travel with liquid velocity otherwise.

Kelvin-Helmholtz instability was thought at one time to be responsible for the generation of waves by wind on open bodies of water. Lamb (1947) points out, however, that for the case of air flow over water, the minimum wind velocity to produce this instability is 646 cm/s, while waves on water are observed at much lower wind speeds. Miles (1959) investigated this problem in considerable detail and reached essentially the same conclusion. He points out that there is another mechanism of wave generation which would become active long before the onset of Kelvin-Helmholtz instability.

This other mechanism, which Miles (1959, 1962a, b) analyzed in a series of papers, is the result of the component of aerodynamic pressure in phase with the wave slope.

This aerodynamic pressure is

$$P_g = \beta\frac{\partial\eta}{\partial x}$$

which may be written as

$$P_g = -\frac{\beta}{c}\frac{\partial\eta}{\partial t} \tag{4}$$

since Miles has shown that, at least for small values of β, this component does not influence the wave speed.

Substituting this in Eq. 1, we have

$$\frac{\rho_1}{k}\frac{\partial^2\eta}{\partial t^2} - \frac{\beta}{c}\frac{\partial\eta}{\partial t} + (\rho_1 g + T_1 k^2)\eta = 0$$

for which Miles (1959) has obtained the solution:

$$\eta = a \exp\left(\frac{\beta k t}{2c\rho_1}\right)\sin k(x - ct) \tag{5}$$

For positive values of β, the waves are unstable, but this instability exhibits itself in a relatively slow growth, while the waves retain their periodic characteristics. While the experimental results (Bole and Hsu, 1967) have shown growth rates usually in excess of those predicted by Miles, it is generally conceded that this is the mechanism of the generation and at least initial growth of waves.

In the two-phase flow case where the gas and liquid flow in a closed channel, the conditions are not significantly different when the interface is smooth. Thus it is unlikely that the Kelvin-Helmholtz instability is responsible for the generation and initial growth of waves; this phenomenon is due to the component of pressure in phase with the wave slope.

As the waves grow, they begin to occupy a significant portion of the channel cross section and the pressure distribution in the gas deviates considerably from that of the open flow. As may be found from the Bernoulli equation, the pressure component 180° out of phase with the wave increases rapidly, and as the waves approach the top wall they must eventually reach the Kelvin-Helmholtz instability. This was proposed by Kordyban and Ranov (1970) to be the cause of the formation of slugs.

Examining the existing information, it appears that slugs form somewhat before one would expect the pressure variation due to a velocity difference between the crest and the trough to be sufficiently high to overcome the effect of gravity. Kordyban and Ranov (1967) examined finite amplitude waves and found that the instability should occur at $ka = 0.9$ while Mishima and Ishii (1980) continuing with this analysis assume the wave steepness, ka, to be 1.0. The height-to-length ratio of such waves would be 0.318, but the steepest waves observed in the channels were found to possess a height-to-length ratio of 0.10.

In applying the one-dimensional Bernoulli equation, Wallis and Dobson (1973) found the pressure amplitude at transition to slug flow to be too low by a factor of two. Taitel and Dukler, having reached essentially the same conclusion, proposed that large waves are less stable. Kordyban (1973, 1980) has measured actual pressure distribution over waves and correlated it by

$$\Delta p = K\frac{\rho_g}{2}(V_c^2 - V_T^2), \tag{6}$$

where K is a constant having a value larger than unity. Even so, the transition to slug flow occurs somewhat earlier than predicted by this relationship.

It should also be noted that stationary waves required by Kelvin-Helmholtz instability were never observed experimentally.

The Kelvin-Helmholtz instability has been applied to the whole wave, but whether it may occur locally, over a small part of the wave, has not been considered. This may best be illustrated by considering that, at the point of instability the upward force due to pressure difference is equal to the downward force of gravity neglecting vertical acceleration.

Referring to Figures 1 and 2, if the liquid surface profile is $\eta =$

$\eta(x)$ and the pressure over liquid is $p_g = p_g(x)$, then balancing the forces on a liquid element at the point of instability, $(\rho_1 - \rho_g)gd\eta dx = -dpdx$ or $-dp/dx/d\eta/dx = (\rho_1 - \rho_g)g$.

The liquid then is particularly susceptible to instability at locations where the profile is small and pressure gradient is high.

For waves in closed channels, the one-dimensional Bernoulli equation may be used to determine the pressure over the wave at any point $p - p_o = (K\rho_g/2)(V_g^2 - V^2)$ where K is used to allow for the effect of velocity distribution.

Differentiating the above equation

$$\frac{dp}{dx} = K\rho_g V \frac{dV}{dx} \tag{7}$$

By continuity

$$Vh = V_g h_g = V(h_g - \eta)$$

$$\frac{dV}{dx} = \frac{V_g h_g}{(h_g - \eta)^2}\frac{d\eta}{dx}$$

and

$$\frac{dp}{dx} = -K\rho_g V \frac{V_g h_g}{(h_g - \eta)^2}\frac{d\eta}{dx},$$

and the instability criterion becomes

$$(\rho_1 - \rho_g)g = K\rho_g V \frac{V_g h_g}{(h_g - \eta)^2}. \tag{8}$$

If this is now applied to the wave approaching the top wall of the channel, it will be noted that the crest will become unstable before the main portion of the wave, since both the velocity V and wave profile η have the highest values there.

In order to establish the existence of this local instability and to determine its form, a series of motion picture photographs was taken. The pictures are described in the following sections.

EXPERIMENTAL

The experimental work was carried out in a rectangular channel made of acrylic plastic 10 cm deep, 15 cm wide, and approximately 7 m long. The downstream end of the channel was equipped with a sloping beach to minimize reflection of the waves. The upstream end contained an air inlet and a wave generator consisting of a vertical plate extending across the channel. The channel was filled with a predetermined amount of water and air was admitted parallel to the water surface. The air flow rate was measured by a laminar flow meter.

Since it is known that slugs originate at random times and locations, it was necessary to devise a technique to control the production of slugs. This was done by generating a solitary wave by a single stroke of the wave maker. To force the slugs to form at a particular location, the downstream end of the channel was lowered slightly. This causes the mean air velocity to *increase* along the channel, eventually reaching the value at which slugs were produced.

To photograph the slugs as they formed, a motion picture camera located approximately 2 m from the channel and was operated at 64 frames per second. The camera was mounted on a carriage which traveled on a track parallel to the wave channel. It was attempted at first to mechanize the travel of the carriage and to control it in such a way that it would always be aligned with the forming slugs, but the controls become complicated and unwieldy and seemed to perform no better than manual aiming and operation of the camera; the latter method was chosen for the experiments.

To avoid the wall effects, the central plane of the channel was photographed. For this purpose, the channel was covered with matte black paper except for the front face and a slit in the center of the top and bottom of the channel; the central portion of the channel was illuminated through this slit. To provide good visual contast between air and water, light-reflecting material, Mearlmaid AA, was added to water.

Before the experiments began, the channel was filled with water to a predetermined level and the air flow rate was adjusted to a value close to the formation of slugs. At this point small air-generated waves were observed on the surface of water. The wave generator was then permitted to move through one stroke, producing a single wave higher and longer than the others. This wave was followed and photographed during its travel in the downstream one third of the channel. With proper adjustment of the air flow rate, a slug could be produced from every high wave; about 60 slugs were photographed.

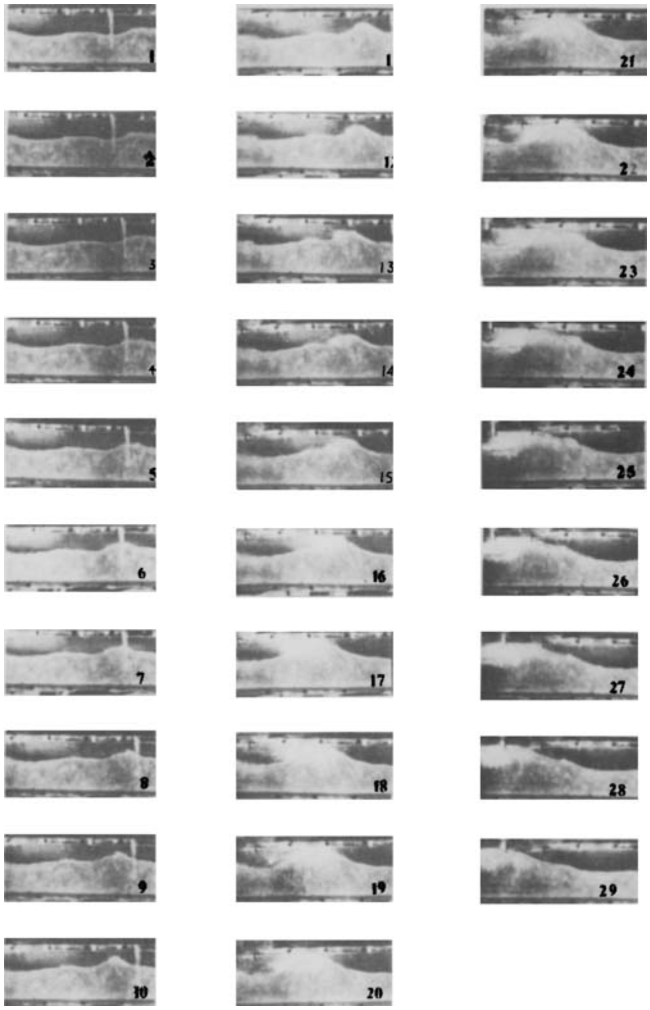

Figure 3. Photographic sequence: history of development of a slug.

RESULTS

Figure 3 shows a series of successive frames depicting a complete history of slug development. In the initial frames, a wave with a smooth profile may be observed which proceeds relatively unchanged along the channel. In frame 4, a small disturbance appears near the crest; it is this disturbance that grows rapidly, encompassing progressively more liquid in the wave. In frame 12, the resulting liquid structure is very steep and then appears to topple over, under the shearing action of the moving air, recovering and reaching the channel top in frame 16. After this, the slug proper develops. This development is typical, but some cases are not as elaborate, with liquid reaching the channel top in several frames.

As will be discussed later, the appearance of the disturbance on

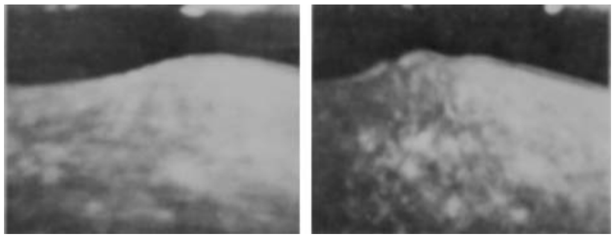

Figure 4. Sudden appearance of wavelets on initially smooth wave crest.

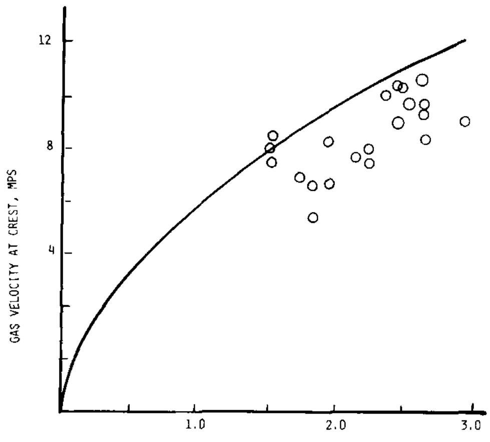

Figure 5. Relationship of velocity at crest, V_c, to distance from crest to top of channel, h_c, just prior to observed instability in waves.

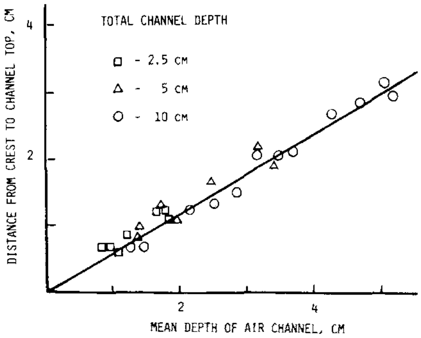

Figure 7. Distance from wave crest to channel top as a function of mean air passage depth for highest waves near the formation of slugs.

the smooth face of the wave is proposed to be the onset of the Kelvin-Helmholtz instability. Therefore, Figure 4 is also presented. It consists of two frames taken about 0.08 sec. apart. The first shows a smooth face of the wave; in the second, small wavelets have suddenly appeared at the crest.

Figure 5 presents the depths of the air passage at the wave crest plotted against mean air velocity at the same location. The measurements were taken from film frames just prior to those showing noticeable surface disturbances. The data are compared to the curve proposed by Kordyban (1973, 1980). An agreement similar to that in the cited work may be observed. This is significant because in the present work slugs were produced in a rather artificial manner, while in previous work the slugs originated from wind-generated waves.

Figure 6 shows the motion of a typical wave plotted against time. The measurements were taken from consecutive frames of the motion picture film and encompass the time from the first disturbance to the point at which the slug formed. Two curves are presented, one showing the motion of the wave trough considered to represent the motion of the whole wave and the second showing the motion of a wavelet at the crest. It will be noted that both have practically the same velocity, with the disturbance being slightly faster, probably due to limited depth of water.

DISCUSSION

Based on observations in this work, the following modification to the mechanism of slug formation is proposed. The faster-moving gas generates the initial waves on the smooth interface by virtue of the pressure component in phase with the wave slope, as proposed by Miles. These waves grow relatively slowly, but as they become larger they block a significant portion of the air passage, producing large pressure variations 180° out of phase with the wave profile. In this manner the waves approach Kelvin-Helmholtz instability. Before the instability of the whole wave can take place however, local instability occurs at the crest because of particularly high pressure gradient and low wave slope there. As a result of this instability, small wavelets form at the crest, as seen in Figures 3 and 4.

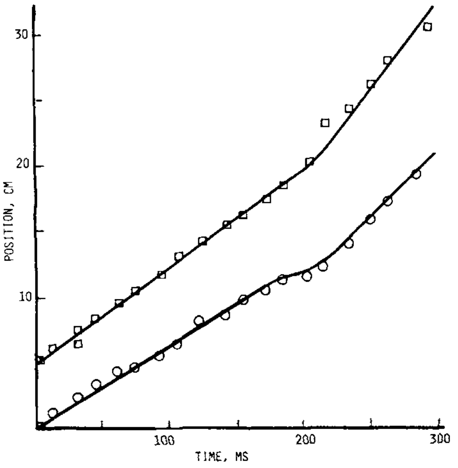

Figure 6. Motion of typical wave after observed instability at crest: O-O motion of wave trough; □-□ motion of disturbance at crest.

Ordinarily such wavelets, having a significantly lower celerity, would travel backward with respect to the main wave, but in this case they move with practically the same celerity as the main wave. Since the liquid particle motion is nearly horizontal at the crest and its velocity is equal to wave celerity for high waves (Kordyban, 1977), the wavelengths may be considered stationary with respect to the water current as required by Kelvin-Helmholtz instability. It is felt then that the presence of this instability has been demonstrated.

Once they are unstable, the wavelets continue to grow until they reach the top of the channel, except when the resulting very steep liquid structure is toppled by the moving gas. It was observed that in some cases such waves returned to stability and never became slugs.

The question now arises as to how this information can be used to predict more accurately the onset of slugging. To make quantitative use of the information it is necessary to know accurately the aerodynamic pressure gradient at the wave crest; however, this is presently unavailable. The numerical value of constant K in Eq. 6 as determined by Kordyban (1973), is an average value for the pressure difference between the crest and trough; its local value at the crest is probably different.

It should also be noted that in the analysis of the stability of the liquid surface at the crest, it is necessary to include the surface tension term, since the wavelets are small and their stability will be affected strongly by the surface tension.

Some estimate of the value of K at the crest may be obtained from the data on the transition to slug flow. Use will be made of the data from Kordyban (1977), since the details on wave heights are available. These data form a rather clean line when plotted using coordinates proposed by Wallis and Dobson (1973), i.e., nondimensional gas velocity j^* vs. the void fraction α. The function can be expressed as

$$j^* = 0.353\alpha^{3/2}, \tag{9}$$

which is similar to the relationship proposed by Wallis and Dobson except that their coefficient is 0.5.

In order to make use of Eq. 8, it is necessary to find a relationship between the void fraction α and the channel. Such a relationship may be observed in Figure 7, where this distance for the highest waves just prior to slugging has been plotted against the mean depth of the air passage h_g for three different total depths. The function can be expressed by a simple relationship

$$h_c = 0.6\, h_g \tag{10}$$

The form of Eq. 8 may be modified by recognizing that

$$q = h_g V_g = V_c h_c$$

and

$$\alpha = \frac{h_g}{H}$$

Substituting Eq. 8 and rearranging

$$\frac{K\rho_q}{\rho_1 - \rho_g} \cdot \frac{q^2}{g\left(\alpha H \cdot \dfrac{h_c}{h_g}\right)^3} = 1$$

but

$$\frac{\rho_g}{\rho_1 g} \cdot \frac{q^2}{gH^3} = j^{*2}$$

and

$$\frac{Kj^{*2}}{\alpha^3 \left(\dfrac{h_c}{h_g}\right)^3} = 1$$

$$j^* = \left(\frac{h_c}{h_g}\right)^{3/2} \cdot \frac{1}{K^{1/2}}\, \alpha^{3/2}.$$

By comparing this to Eq. 9,

$$K = \left(\frac{h_c}{h_g}\right)^3 (0.353)^2$$

Using the value of (h_c/h_g) from Eq. 10, the numerical value of K becomes $K = 1.80$.

This value, although not unreasonable, is probably somewhat high, because the waves considered in cited work were the highest waves just prior to the formation of slugs and it was the next higher wave that produced the slug.

The results of the present work then agree reasonably well with the published data on the transition to slug flow. They present a somewhat different explanation of the Wallis-Dobson relationship and provide a firmer theoretical basis for the statement of Taitel and Dukler that larger waves are more unstable.

These tests were limited to air and water. It is of practical interest to consider possible changes in this mechanism if other fluids were used. Viscosity does not enter the analytical considerations in this work and it is assumed that the results should be valid for a broad band of viscosities. Surface tension, on the other hand, tends to stabilize the liquid surface. For a liquid with smaller surface tension this mechanism should be still valid, but for greater surface tension it is possible that the instability of the whole wave would occur before the formation of the wavelets at the crest.

NOTATION

a = wave amplitude
c = wave celerity
g = acceleration of gravity
H = total depth of channel
h_c = distance from wave crest to channel top
h_g = mean depth of air passage
j^* = dimensionless gas velocity
k = wave number
K = constant, defined in text
p = pressure
P_g = gas pressure
P_o = reference pressure
Δp = pressure difference between crest and trough
q = volumetric flowrate per unit width
t = time
T = surface tension
V = velocity
V_c = gas velocity at wave crest
V_g = mean gas velocity
V_t = gas velocity at wave trough

Greek Letters

α = void fraction
α_g = constant defined in text
β = constant defined in text
η = wave profile
ρ_g = gas density
ρ_1 = liquid density

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